

Beyond $\eta/s = 1/4\pi$

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Beyond $\eta/s = 1/4\pi$

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ABSTRACT: We use a low-energy effective description of gauge theory/string theory duality to argue that the Kovtun-Son-Starinets viscosity bound is generically violated in superconformal gauge theories with non-equal central charges $c \neq a$. We present new examples (of string theory constructions and of gauge theories) where the bound is violated in a controllable setting. We consider the comparison of results from AdS/CFT calculations to the QCD plasma in the context of this discussion.

KEYWORDS: Gauge-gravity correspondence, AdS-CFT Correspondence

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1 Introduction

Over the past decade, the AdS/CFT correspondence [1, 2] has been developed to provide a powerful tool to investigate the thermal and hydrodynamic properties for certain strongly coupled gauge theories [3]. At the same time, recent experimental results from the Relativistic Heavy Ion Collider (RHIC) have revealed a new phase of nuclear matter, known as the strongly coupled quark-gluon plasma (sQGP) [4]. Recently, there has been great interest in possible connections between these two advances, in particular, using the AdS/CFT to gain theoretical insight into the sQGP [5]. The primary motivation for this possible connection is the observation that a wide variety of holographic theories exhibit an exceptionally low ratio of shear viscosity to entropy density $\eta/s = 1/4\pi$ while the RHIC data seems to indicate that this ratio is unusually small for the sQGP and even seems yield roughly $\eta/s \sim 1/4\pi$ [6].

Motivated by the results from the AdS/CFT correspondence, Kovtun, Son and Starinets (KSS) proposed a now celebrated bound for the viscosity-to-entropy-density ratio [7]. That is, for all fluids in nature the ratio η/s is bounded from below:

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}. \quad (1.1)$$

This bound certainly appears to be satisfied by all common substances observed in nature [8]. Using the AdS/CFT correspondence, the bound has been shown to be saturated in all gauge theories in the planar limit and at infinite 't Hooft coupling (with various gauge groups, matter content, with or without chemical potentials for conserved U(1) charges, with non-commutative spatial directions, in external background fields) that allow for a dual supergravity description [8–17]. The bound is not saturated but it is still satisfied in all four-dimensional¹ conformal gauge theories with equal a and c central charges, again allowing for a string theory dual and in the planar limit and with large but finite 't Hooft coupling [19, 20].

One may ask if the KSS bound (1.1) is indeed of fundamental importance to nature? However, the answer appears to be “no”. It was pointed out by [21] that the bound is violated in a nonrelativistic gas with increasing number of species and by [22–24], that it can be violated in effective theories of higher derivative gravity. Of course, the true question is whether or not the violation occurs in quantum field theories that allow for a consistent ultraviolet completion [25]. In fact, Kats and Petrov [24] proposed an explicit example of a gauge theory/string theory duality where a violation of the KSS bound occurs in a controllable setting — see also [26]. However, one may easily draw into question the veracity of this claim.

In particular, the calculations in [24] were presented in terms of an effective five-dimensional gravity theory. However, the proposed duality is between a gauge theory and a ten-dimensional string theory. Thus, it seems the gravity calculations should be performed within the full ten-dimensional string theory background constructed to required order in α' . Alternatively, beginning with the ten-dimensional background, one could carefully perform the Kaluza-Klein reduction but this would require keeping track of all of the fields and their interactions in the effective five-dimensional theory. For instance, the reduction may produce scalar fields which it seems are likely to effect the calculations at the order to which they must be performed to detect the potential violation of the viscosity bound.

Our primary motivation for the present work was to examine in detail the claimed violation of the viscosity bound (1.1) in [24]. In fact, we are able to sharpen the arguments in terms of an effective five-dimensional gravity dual and confirm that the KSS bound will be violated as long as the central charges of the conformal gauge theory satisfy a number of conditions: $c \sim a \gg 1$ and $|c - a|/c \ll 1$ are required to guarantee the reliability of the low energy effective action and then the inequality

$$c - a > 0, \quad (1.2)$$

¹Preliminary analysis indicates that the bound is satisfied under the same conditions in three-dimensional conformal gauge theories [18].

produces a violation of the KSS bound [22, 24].

An outline of the paper is as follows: In section 2, we examine in detail when an effective five-dimensional gravity dual yields a reliable description of the superconformal gauge theory. In section 3 we compute $(c - a)$ in variety four-dimensional superconformal gauge theories. This produces new examples where we can reliably state that the KSS bound is violated. Given these observations, we consider the comparison of results from AdS/CFT calculations to the sQGP in section 4. Finally, we provide a concluding discussion in section 5. Appendix A elaborates on the discussion of field redefinitions in the presence of other bulk fields, while appendix B provides an explicit realization of our effective AdS/CFT duality in a stringy context where ten-dimensional supergravity plus probe branes is a reliable approximation.

2 Effective description of conformal gauge theory/string theory duality

According to the AdS/CFT correspondence [2], any four-dimensional superconformal gauge theory will have a dual description in terms of quantum gravity with a negative cosmological constant in five dimensions. Now for particular cases where it is sensible to consider the conformal gauge theory with large- N_c and strong coupling, our intuition is that the dual description is well approximated by Einstein gravity in a five-dimensional AdS spacetime. In this framework, higher curvature (or more broadly higher derivative) interactions are expected to arise on general grounds, e.g., as quantum or stringy corrections to the classical action. Hence a more refined description will be given by an effective action where the cosmological constant and Einstein terms are supplemented by such higher curvature corrections. Here we consider when such an effective action approach yields a reliable description of the superconformal gauge theory.

A key assumption in our discussion will be that:

The effective five-dimensional gravity theory is described by a sensible derivative expansion. That is, we expect that the higher curvature terms are systematically suppressed by powers of the Planck length, ℓ_P .

Hence we can expect the effective gravity action in five dimensions to leading order to take the form

$$I = \frac{1}{2\ell_P^3} \int d^5x \sqrt{-g} \left[\frac{12}{\tilde{L}^2} + R + \tilde{L}^2 \left(\tilde{\alpha}_1 R^2 + \tilde{\alpha}_2 R_{ab} R^{ab} + \tilde{\alpha}_3 R_{abcd} R^{abcd} \right) + \dots \right], \quad (2.1)$$

where the scale \tilde{L} will correspond to the AdS curvature scale, at leading order, and we assume that $\tilde{L} \gg \ell_P$. We have parameterized the curvature squared couplings with the AdS curvature scale, as is convenient for explicit calculations, but we expect that the dimensionless couplings $\alpha_i \sim \ell_P^2/\tilde{L}^2 \ll 1$ in accord with our assumption of a sensible derivative expansion. Further, compared to these interactions, the six- and higher derivative terms, which have been left implicit, are suppressed by further powers of ℓ_P^2/\tilde{L}^2 . For example, an interaction of the form $\lambda \tilde{L}^4 R R_{abcd} R^{abcd}$ would have $\lambda \sim \ell_P^4/\tilde{L}^4 \ll \tilde{\alpha}_i$.

At this point, we note that we can simplify the form of the action by making a field redefinition $g_{ab} \rightarrow g_{ab} + \delta g_{ab}$ with [22, 24]

$$\delta g_{ab} = \frac{8}{3} (5\alpha_1 + \alpha_2) g_{ab} + \alpha_2 \tilde{L}^2 R_{ab} - \frac{1}{3} (2\alpha_1 + \alpha_2) \tilde{L}^2 R g_{ab}, \quad (2.2)$$

which then simplifies the action to

$$I = \frac{1}{2\ell_p^3} \int d^5x \sqrt{-g} \left[\frac{12}{L^2} + R + \alpha_3 L^2 R_{abcd} R^{abcd} + \dots \right]. \quad (2.3)$$

The implicit terms implied by the ellipsis all contain six or more derivatives suppressed by at least ℓ_p^4/\tilde{L}^4 , as described above. Hence, the field redefinition (2.2) has succeeded in eliminating the R^2 and $R_{ab}R^{ab}$ terms.² This makes clear that, at this order, the gravity action contains two and only two dimensionless small parameters: ℓ_p/L and α_3 .

We return to this point after making a number of observations: first, given the effective action (2.3), we might consider making a further field redefinition of the form

$$\delta g_{ab} = \lambda_1 L^4 R_{acde} R_b{}^{cde} + \lambda_2 L^4 g_{ab} R_{cdef} R^{cdef}, \quad (2.4)$$

which would modify the action by adding terms of the form

$$\delta I = \frac{1}{2\ell_p^3} \int d^5x \sqrt{-g} \left[6L^2 (\lambda_1 + 5\lambda_2) R_{abcd} R^{abcd} - L^4 \lambda_1 R^{ab} R_{acde} R_b{}^{cde} + \frac{L^4}{2} (\lambda_1 + 3\lambda_2) R R_{abcd} R^{abcd} \right]. \quad (2.5)$$

Hence, given the first term above, it would seem that we can use these field redefinitions to remove the α_3 term in (2.3). Note that the latter would require that $\lambda_{1,2} \sim \ell_p^2/L^2$ and hence the six-derivative terms, appearing in the second line of (2.5), would only be suppressed by this same factor ℓ_p^2/L^2 . However, our assumption is that the derivative expansion organizes the effective action so that any such term is suppressed by a factor of ℓ_p^4/L^4 . Hence if we wish to maintain this structure, then we must require that $\lambda_{1,2} \sim \ell_p^4/L^4$ and so this field redefinition could only make higher order corrections to α_3 .

Next, we observe that with the original field redefinitions (2.2), Newton's constant (i.e., the coefficient of the Einstein term) has been kept fixed but the curvature scale L has to be redefined as

$$L^2 = \tilde{L}^2 \left(1 - \frac{20}{3} (5\tilde{\alpha}_1 + \tilde{\alpha}_2) + \dots \right). \quad (2.6)$$

In principle, the coupling $\tilde{\alpha}_3$ was also corrected with $\alpha_3 = \tilde{\alpha}_3 + O(\tilde{\alpha}_1^2, \tilde{\alpha}_2^2, \tilde{\alpha}_1\tilde{\alpha}_2)$. However, we do not specify the latter in detail, as it actually requires specifying the field redefinition (2.2) more precisely, i.e., to order $\tilde{\alpha}_i^2$. But these expressions do illustrate the point that in general the parameters in this effective action (2.3) may be complicated functions of the microscopic parameters of the quantum gravity theory. For example, in a string

²Of course, the coefficients of these two interactions could be tuned to any convenient values. For example, this would allow us to assemble the curvature-squared terms to be the square of Weyl-curvature or the Gauss-Bonnet term [22], either of which may be advantageous for certain calculations.

or M-theory framework, they would arise upon the Kaluza-Klein compactification of the higher dimensional geometry and these low energy parameters would depend on all of the details for the compactification. In general, we would also expect that these parameters also receive quantum ‘corrections’, which might in turn include both perturbative and nonperturbative contributions.

Note, however, that the dual theory is assumed to be dual to a four-dimensional conformal field theory. Hence at any order in the derivative expansion, the gravity theory admits a five-dimensional anti-de Sitter vacuum, although the precise characteristics, i.e., curvature, of the latter may change as we increase the accuracy of our calculations. Given the action (2.3), the curvature of the AdS space is:

$$\hat{L}^2 = L^2 \left(1 - \frac{2}{3} \alpha_3 + \dots \right). \quad (2.7)$$

Again, this curvature is dependent on the microscopic details of the quantum gravity theory.

The key observation, which we review here, is that the two dimensionless parameters identified above are simply related to parameters characterizing the dual CFT. First, we recall that the conformal anomaly of a four-dimensional CFT can be identified by putting the theory in a curved spacetime and observing [27]

$$\langle T^\mu{}_\mu \rangle_{CFT} = \frac{c}{16\pi^2} I_4 - \frac{a}{16\pi^2} E_4. \quad (2.8)$$

Here c and a are the two central charges of the CFT and E_4 and I_4 correspond to the four-dimensional Euler density and the square of the Weyl curvature, respectively. Explicitly,

$$E_4 = R_{\mu\nu\rho\lambda} R^{\mu\nu\rho\lambda} - 4R_{\mu\nu} R^{\mu\nu} + R^2, \quad I_4 = R_{\mu\nu\rho\lambda} R^{\mu\nu\rho\lambda} - 2R_{\mu\nu} R^{\mu\nu} + \frac{1}{3} R^2. \quad (2.9)$$

Holographic techniques allow precisely the same expression to be calculated with the result [28–30],

$$\langle T^\mu{}_\mu \rangle_{holo} = -\frac{\hat{L}^3}{16\ell_p^3} (E_4 - I_4) + \frac{\hat{L} L^2}{4\ell_p^3} \alpha_3 (E_4 + I_4), \quad (2.10)$$

Hence comparing (2.8) and (2.10), we arrive at

$$\frac{L^3}{\ell_p^3} \simeq \frac{c}{\pi^2} \left(1 - \frac{3}{8} \frac{c-a}{c} \right), \quad \alpha_3 \simeq \frac{1}{8} \frac{c-a}{c}. \quad (2.11)$$

In these expressions, we have used our assumption of a sensible derivative expansion, which dictates that $\alpha_3 \ll 1$.

One conclusion then is that if we require the quantum gravity theory is described by a low energy action with sensible derivative expansion, we are restricted to consider CFT’s for which

$$c \sim a \gg 1 \quad \text{and} \quad |c-a|/c \ll 1. \quad (2.12)$$

Further, the effective action is expected to contain further higher curvature terms and the dimensionless coefficients appearing in these interactions would be related to new parameters characterizing the CFT — for example, see [31]. Our assumption of a sensible

derivative expansion then restricts the size of these parameters, i.e., the CFT's of interest should have these parameters being proportional to inverse powers of the central charge c .

Above, we observed that the AdS/CFT correspondence dictates the values of the leading parameters in the effective gravity action terms of the central charges of the dual CFT according to (2.11). Hence if the central charges of the CFT are known (and the inequalities (2.12) are satisfied), we can be confident of the precise form of this effective action (2.3) to leading order, even if we do not understand the microscopic details underlying the quantum gravity theory. Then if we are careful to respect the limitations of the derivative expansion, we can work reliably with the gravity action (2.3) to determine the properties of the CFT using the standard AdS/CFT correspondence. Note that only the dimensionless ratios in (2.11), but not the Planck length ℓ_P , appear in any physical results for the CFT. Of course, this is in accord with the fact that for a supersymmetric CFT, supersymmetry combines with diffeomorphism and conformal invariance to completely dictate the form of the two- and three-point correlators of the stress-energy tensor in terms of these two central charges, a and c [33]. Hence while we can reproduce these correlators with the dual gravity action (2.3), the latter also allows us to calculate more interesting properties, such as thermal transport coefficients of the CFT. One interesting example is the shear viscosity [22, 24]

$$\frac{\eta}{s} = \frac{1}{4\pi} (1 - \delta + \dots), \tag{2.13}$$

where we have introduced

$$\delta \equiv \frac{c - a}{c} = 8\alpha_3 + \dots \tag{2.14}$$

Hence the sign of δ in the CFT or of the $R_{abcd}R^{abcd}$ term in effective gravity action determines whether or not the viscosity bound (1.1) is respected or violated at this order. In particular, the bound is violated if $c > a$.

Of course, according to the standard AdS/CFT dictionary, the metric is dual to the stress-energy tensor of the CFT and so with the gravity action (2.3), we are restricted to study the properties of this one operator. In general, we should expect the full CFT will have a spectrum of interesting operators, possibly including a variety of relevant, irrelevant and marginal operators. The latter would then be dual to other fields which may also play an interesting role in the gravity theory. Hence our preceding conclusions may seem somewhat naive since we have restricted the discussion to the pure gravity sector of the theory. Therefore we must show that such operators do not effect our conclusions.

As an example, consider the case where the gravity theory that contains a number of scalars ϕ^k . As above, we assume that the effective gravity theory is described by a sensible derivative expansion. In principle, a large number of four-derivative terms could appear in the effective action but as described in appendix A, field redefinitions can be used to greatly simplify the action. The final action can be written as

$$I = \frac{1}{2\ell_P^3} \int d^5x \sqrt{-g} \left[\mathcal{U}(\phi^m) + R - \mathcal{K}_{ij}(\phi^m) \nabla\phi^i \cdot \nabla\phi^j + \mathcal{A}_3(\phi^m) R_{abcd}R^{abcd} + \mathcal{B}(\phi^m, \nabla_a\phi^m, \nabla_a\nabla_b\phi^m) + \dots \right]. \tag{2.15}$$

While more details are provided in the appendix, \mathcal{B} combines the remaining four-derivative interactions which explicitly contain derivatives of the scalar fields. An important point is that all of these interactions contain at least two factors of scalar derivatives. Then, since we are treating these terms perturbatively within the derivative expansion and the scalars will be constant in the leading solutions of interest, they remain constant at the next order. Hence we may ignore these terms for the remainder of the discussion.

In describing the rest of the terms in (2.15), we should begin by saying that we have adopted the convenient (supergravity) convention where the scalar fields ϕ^i are dimensionless. Below, we argue that the scalars vanish in the AdS₅ vacuum and so we may assume that all of the expressions in the action are nonsingular at $\phi^i = 0$. Hence we can express each of the coefficient functions in terms of a Taylor series:

$$\mathcal{U}(\phi^m) = \frac{12}{L^2} \left(1 + u_i \phi^i + u_{ij} \phi^i \phi^j + u_{ijk} \phi^i \phi^j \phi^k + \dots \right), \quad (2.16)$$

$$\mathcal{K}_{ij}(\phi^m) = k_{ij} + k_{ijk} \phi^k + k_{ijkl} \phi^k \phi^l + k_{ijklm} \phi^k \phi^l \phi^m + \dots, \quad (2.17)$$

$$\mathcal{A}(\phi^m) = L^2 \left(\alpha_3 + a_i \phi^i + a_{ij} \phi^i \phi^j + a_{ijk} \phi^i \phi^j \phi^k + \dots \right). \quad (2.18)$$

Now in keeping with our assumption of the derivative expansion above, a second key assumption here is that:

All of the coupling coefficients in each of (2.16), (2.17) and (2.18) above are of the same order (with the exception of u_i).

That is, all of the couplings $u_{ij\dots}$ in (2.16) and $k_{ijk\dots}$ in (2.17) may be of order one (or higher order in ℓ_p^2/L^2), with the exception of u_i – which we address below. Similarly, α_3 and all of the subsequent couplings $a_{ij\dots}$ in (2.18) are assumed to be of order ℓ_p^2/L^2 (or higher). Of course, each of these couplings may in general be a complicated function of ℓ_p^2/L^2 and so here we are demanding that α_3 and $a_{ij\dots}$ do not have order one (or order ℓ_p/L) contributions. Within this framework, the corresponding scalar masses are of the order of the AdS curvature scale, i.e., $m_k^2 \sim 1/L^2$. Hence each of the dual scalar operators \mathcal{O}_k has a conformal dimension of order one. These operators may be relevant, irrelevant or marginal. An exactly marginal operator is an exceptional case, which will receive detailed consideration below. As before, the ellipsis in (2.15) corresponds to six- and higher derivative terms which implicitly are suppressed at least by couplings of order ℓ_p^4/L^4 , as in the previous discussion.

The dual theory is a conformal field theory, which again implies that at any order in the derivative expansion, the gravity theory (2.15) admits an AdS₅ vacuum. Further, in the conformal vacuum, the expectation value of any of the operators must vanish, i.e.,

$$\langle \mathcal{O}_k \rangle_0 = 0, \quad \text{as well as} \quad \langle T_{\mu\nu} \rangle_0 = 0. \quad (2.19)$$

This property is reflected in the gravity theory with the vanishing of the dual scalar fields in the AdS₅ vacuum. A possible exception to this conclusion arises with an exactly marginal operator. In principle, the corresponding massless scalar in the dual gravity theory can take on any constant value. However, we will define this expectation value of the scalar field to be zero for the vacuum that we are studying here.

Let us now turn to the special case of the exceptional couplings u_i . For the AdS₅ space to be a solution with $\phi^i = 0$ at leading order in the derivative expansion, i.e., dropping the curvature-squared and higher order terms, it must be true that $u_i = 0$ at this order. However, when curvature-squared term is included, the scalar equations of motion yield

$$\left[\frac{\delta \mathcal{U}}{\delta \phi^i} + \frac{\delta \mathcal{A}}{\delta \phi^i} R_{abcd} R^{abcd} \right]_{\phi^k=0} = \frac{12}{L^2} u_i + L^2 a_i \frac{40}{L^4} = 0. \quad (2.20)$$

assuming an AdS₅ background with vanishing scalars. Hence we find that consistency demands that the scalar potential contains linear couplings of order ℓ_p^2/L^2 :

$$u_i = -\frac{10}{3} a_i. \quad (2.21)$$

An alternate interpretation would be that if we set $u_i = 0$ then the AdS₅ solution is stable to leading order but in general the appearance of the curvature-squared term will then cause the scalars to acquire an expectation value of order ℓ_p^2/L^2 in the vacuum. We are simply redefining the scalars to absorb this constant shift in our approach.³

Although the scalars vanish in the AdS₅ vacuum, one can expect that the curvature-squared term will source the various scalar fields in more general backgrounds. However, one would still have that $\phi^k \sim a_k \sim \ell_p^2/L^2$ in such a background. We must consider two particular examples in our discussion. The first relevant example would be a black hole background and this effect implies that in a thermal bath the dual operators acquire expectation values $\langle \mathcal{O}_k \rangle \sim a_k$. The second relevant case comes from the holographic calculation of the conformal anomaly (2.10). While the precise background is typically not specified in these calculations, implicitly, one must be working with more general backgrounds where, in particular, the Weyl curvature is nonvanishing. Hence we must argue that even though the scalars may have nontrivial profile at order ℓ_p^2/L^2 , this will not affect the holographic calculations of the conformal anomaly or the thermal behaviour of the CFT. With a careful consideration below, we will show that the nontrivial scalars can only modify the results at order ℓ_p^4/L^4 . Our general argument was originally formulated in a slightly different context in [11].

As an explicit example, let us consider the calculation of shear viscosity [3, 34, 35]. A key step would be calculating the effective quadratic action for the various graviton fluctuations, i.e., the shear, sound and transverse modes, in the black hole background. The nontrivial scalars can effect these modes in two ways. First they explicitly appear in the action. However, contributions of terms quadratic or higher powers in ϕ^i would be suppressed by ℓ_p^4/L^4 or higher powers. There are two possible sets of linear terms in $\mathcal{U}(\phi^m)$ and in $\mathcal{A}(\phi^m)$ but, as discussed above, the couplings for both of these are already

³An alternate approach would be to use the freedom of field redefinitions so that the square of the Weyl tensor, rather than of the Riemann tensor, appears in the effective action (2.15). Then because the AdS vacuum has vanishing Weyl curvature, the scalar equations of motion would be unaffected by the curvature-squared term and u_i would remain zero at this order. Note that in this approach, the AdS curvature would also match precisely the scale L appearing in the action. However, the additional $R_{ab}R^{ab}$ and R^2 interactions, appearing in the Weyl-curvature squared, would modify the holographic anomaly (2.10) in precisely such a way to reproduce the same expressions as in (2.11).

order ℓ_{p}^2/L^2 and so they only contribute with an overall factor of ℓ_{p}^4/L^4 . Secondly, the nontrivial scalars will modify the background geometry through Einstein's equations, but similar reasoning shows that the modifications of the metric would again be order ℓ_{p}^4/L^4 . Hence, even though the scalars themselves appear at order ℓ_{p}^2/L^2 , their effect is only felt by the graviton modes at order ℓ_{p}^4/L^4 . Hence the calculation of η/s can be reliably made at order ℓ_{p}^2/L^2 while ignoring all of the scalar fields, i.e., with the effective gravity action (2.3). The same general argument applies to calculations of other thermal properties from the black hole background or of the holographic conformal anomaly.

Next, we make a few comments on the extension of our discussion to include vectors in the gravity theory — see also appendix A. If we consider some number of Abelian gauge fields, the vectors are dual to conserved currents and the corresponding U(1) gauge symmetries are identified with global symmetries in the CFT [2]. A complete discussion of the contributions of these gauge fields to the four-derivative gravitational action would be quite lengthy and equally tedious and so we only remark on salient points. First, we restrict the discussion to having only constant gauge fields at leading order in the background. That is, we are only considering the case of vanishing chemical potentials. Next, it is relatively easy to show that the majority (i.e., all but one) of the new four-derivative interactions are at least quadratic in the field strengths of these gauge fields. Hence an argument similar to that below (2.15) applies here as well, with the conclusion that these terms are irrelevant at this order, as long as we are considering backgrounds where the vectors are constant. However, given a set of U(1) gauge fields A_a^i , there is one four-derivative coupling which cannot be dismissed by this argument, namely,

$$I' = \frac{1}{2\ell_{\text{p}}^3} \int L^2 d_i A^i \wedge R^a{}_b \wedge R^b{}_a. \tag{2.22}$$

In keeping with the derivative expansion, these terms, which are linear in the gauge fields, are characterized by a set of dimensionless constants $d_i \sim \ell_{\text{p}}^2/L^2$. Note that we require that under local gauge transformations, I' only produces a surface term and so even if the gravity theory contains scalars, we cannot replace the constants d_i by general functions $\mathcal{D}_i(\phi^m)$. An interaction of this form plays an interesting role in describing the anomaly for the $U(1)_R$ current in supersymmetric CFT's [36, 37]. In fact, in the context of $\mathcal{N} = 2$ gauged supergravity, supersymmetry connects this interaction (2.22) to an $R_{abcd}R^{abcd}$ term [38, 39]. Since this interaction is linear in the gauge potential, it will induce a nontrivial profile in a background where $R^a{}_b \wedge R^b{}_a$ is nonvanishing. However, this combination of curvatures vanishes both for the AdS₅ vacuum and an AdS₅ black hole background and so no profile is induced for these backgrounds. Of course, this result is in keeping with the intuition that a finite charge density is not induced by introducing a finite temperature alone. We should also consider the nontrivial backgrounds implicit in calculating the holographic conformal anomaly (2.10). In general, we expect that a nontrivial profile can be induced for the gauge potentials in this case but we would still only find that $A^k \sim d_k \sim \ell_{\text{p}}^2/L^2$ in such a background. Hence following arguments analogous to those presented to dismiss the effect of nontrivial scalar profiles, we would again find that the nontrivial gauge potentials can only modify the anomaly calculation at order ℓ_{p}^4/L^4 . Therefore the calculations of

both the thermal properties and of the conformal anomaly would remain unaffected by the appearance of additional vector fields. Hence our conclusion once again is that these calculations can be reliably made at order ℓ_p^2/L^2 with the effective gravity action (2.3), while ignoring any matter fields in the gravitational theory.⁴

In closing this section, we return to the special case of an exactly marginal (scalar) operator. As already mentioned above, the dual scalar field ϕ^M is precisely massless and so it can in principle be set to any arbitrary value in the AdS₅ vacuum. This property implies special relations between the couplings for ϕ^M in the effective action (2.15), i.e.,

$$\left(\frac{\delta}{\delta\phi^M}\right)^n \left[\mathcal{U}(\phi^k) + \mathcal{A}(\phi^k) \frac{40}{L^4} \right] \Big|_{\phi^k=0} = 0, \tag{2.23}$$

where $40/L^4$ corresponds to $R_{abcd}R^{abcd}$ in AdS₅, as in (2.20). Unfortunately, these relations make the discussion somewhat more complicated than necessary. So instead, we make a field redefinition such that the effective action (2.15) takes the form

$$I = \frac{1}{2\ell_p^3} \int d^5x \sqrt{-g} \left[\tilde{\mathcal{U}}(\phi^m) + R - \tilde{\mathcal{K}}_{ij}(\phi^m) \nabla\phi^i \cdot \nabla\phi^j + \tilde{\mathcal{A}}_3(\phi^m) C_{abcd}C^{abcd} + \dots \right], \tag{2.24}$$

where C_{abcd} is the Weyl curvature in five dimensions. Since the Weyl curvature vanishes in the AdS₅ vacuum, the only restriction is that the scalar potential $\tilde{\mathcal{U}}(\phi^m)$ is completely independent of ϕ^M . Note, however, that in general $\tilde{\mathcal{A}}_3(\phi^m)$ remains a function of ϕ^M without any restrictions. Hence, even though the couplings in $\tilde{\mathcal{A}}_3$ are naturally of order ℓ_p^2/L^2 as in (2.18), this suppression could be overcome if the massless scalar has a very large expectation value, i.e., $\tilde{\mathcal{A}}_3 \sim \mathcal{O}(1)$ for large ϕ^M . More generally, since ϕ^M can become arbitrarily large, it can produce effective coupling coefficients for the higher derivative terms which are not suppressed as we initially assumed. Hence, our assumption of a sensible derivative expansion will implicitly restrict us to a limited region of the parameter space.

Of course, it is generally expected that a CFT with exactly marginal operators will be an exceptional case and in the absence of exactly marginal operators, these considerations are not required. However, this issue naturally arises in many string realizations of the AdS/CFT correspondence where the dilaton, i.e., the string coupling, is dual to an exactly marginal operator.

In particular, this is the case for the string theory construction which Kats and Petrov [24] suggested produces a violation the KSS viscosity bound (1.1). In this context, the gravitational theory is Type IIb string theory on a AdS₅ × S⁵/Z₂ background, which can be viewed as the decoupling limit of N_c D3-branes overlapping with a coincident collection of four D7-branes and an O7-brane [40]. The dual CFT is four-dimensional $\mathcal{N} = 2$ Sp(N_c) super-Yang-Mills coupled to 4 hypermultiplets in the fundamental representation and 1 hypermultiplet in the antisymmetric representation. The central charges for this gauge theory are [42]:

$$c = \frac{N_c^2}{2} + \frac{3N_c}{4} - \frac{1}{12}, \quad a = \frac{N_c^2}{2} + \frac{N_c}{2} - \frac{1}{24}. \tag{2.25}$$

⁴Of course, the interesting question of corrections in the presence of a chemical potential would require a detailed analysis of the higher order gauge field interactions.

Now for large (but finite) N_c , the central charges satisfy both of the inequalities in (2.12) and so it seems that we can confidently apply the results calculated from the five-dimensional effective action (2.3) with the gravitational couplings fixed by (2.11). Further, as noted by [24], $c > a$ and so the shear viscosity (2.13) is

$$\frac{\eta}{s} = \frac{1}{4\pi} \left(1 - \frac{1}{2N_c} + \dots \right), \tag{2.26}$$

which violates KSS bound (1.1).

However, before accepting this result, we must first consider that in this construction, the string coupling g_s remains a free parameter. That is, this corresponds to the case of an exactly marginal operator which is dual to the dilaton. As usual then, the results for the CFT can be considered in a double expansion in both inverse powers of N_c and of the 't Hooft coupling λ . Alternatively, we can think that the corrections to effective gravity action are governed by two independent scales: the Planck length ℓ_P and the string length ℓ_s . Hence, we must make sure that the higher curvature corrections beyond those explicitly shown in (2.3) are sufficiently suppressed according to the assumed derivative expansion. As we discuss in section 4, there will be no curvature cubed interaction. There is a universal term quartic in curvatures which appears in any (closed) superstring theory [41]. It is known that this term corrects the ratio of viscosity-to-entropy-density at $\mathcal{O}(\lambda^{-3/2})$ [10, 19, 43]. Hence as noted in [24], in order for the correction in (2.26) to dominate, we must have

$$\frac{1}{N_c} \gg \lambda^{-3/2} \quad \Rightarrow \quad \lambda \gg N_c^{2/3}. \tag{2.27}$$

The full correction to η/s from the R^4 term also contains a contribution at $\mathcal{O}(\lambda^{1/2}/N_c^2)$, as well as various nonperturbative corrections [11, 20]. While the latter play no role in the present discussion, formally requiring the first correction to be subdominant yields $\lambda^{1/2} \ll N_c$. While the previous interaction can be associated with the closed string sector, one might also ask if the calculations could be significantly effected by R^4 interactions induced by the branes. As explained in appendix B, such higher curvature terms will be subdominant in the derivative expansion. In particular, a D7-brane induced R^4 term would be accompanied by an additional suppression factor of $g_s V_3 \ell_s^2 / V_5$. In the present case with $V_3 \sim L^3$ and $V_5 \sim L^3$, such an R^4 interaction would only contribute corrections at order $1/(\lambda N_c)$. The final conclusion is that the Kats and Petrov result (2.26) calculated with a five-dimensional effective action (2.3) is reliable within a certain parameter regime (2.27) and that we have at least limited violations of the KSS bound (1.1) in string theory.

In the above string theory example, the curvature-squared term can be associated with the world-volume action of the D7-branes [37, 42]. In appendix B, we have added a discussion which provides a schematic understanding of the origin of this term.

Note the requirement (2.27) is compatible with conventional restrictions implicit in considering the classical gravity limit of the AdS/CFT correspondence in string theory. That is, we have $1 \ll \lambda \ll N_c$ from requiring $\ell_s^2/L^2 \ll 1$ to minimize stringy contributions in the derivative expansion and $g_s \ll 1$ to minimize string loop contributions. While the derivative expansion, and hence $\lambda \gg 1$, is central to the present effective action approach,

there is no need to give a separate account of loop contributions. That is, using the five-dimensional effective action (2.3) did not require a detailed understanding of the underlying microscopic origin of each of the couplings in the full quantum gravity theory. Rather we advocated that if the CFT central charges were given, we could use the AdS/CFT dictionary to fix the gravitational couplings according to (2.11). Of course, consistency also required that these central charges satisfy the inequalities given in (2.12). With this approach, there is no reason that we could not consider the above or other string theory constructions where the string coupling is strong, i.e., $g_s \sim 1$, which implies that $\lambda \sim N_c$ or $\ell_s \sim \ell_P$. In particular, we can apply this approach to evaluate the thermal behaviour of CFT's holographically described by the F-theory constructions of [42]. The case considered by Kats and Petrov corresponds to one of these constructions and in fact, it is the only case with a marginal coupling. In the remaining cases, there are no marginal couplings and the string coupling is pinned at $g_s \sim 1$. Further, as discussed in the following section, with large (but finite) N_c , the central charges again satisfy the inequalities in (2.12) and so the shear viscosity (2.13) yields new violations of the KSS bound (1.1) since $c > a$ in each of these cases.

3 $(c - a)$ in superconformal gauge theories

From the results of the previous section, we can conclude that if the central charges of a four-dimensional superconformal gauge theory satisfy the two inequalities in (2.12), then we can reliably describe the theory with a gravity dual with a five-dimensional effective action (2.3) in which the gravitational couplings fixed by (2.11). Further, the shear viscosity is given by (2.13) and the superconformal theory will violate the KSS bound (1.1) provided that

$$\delta = \frac{c - a}{c} > 0. \tag{3.1}$$

Hence in this section, we explore the central charges of superconformal gauge theories based on simple Lie groups with various matter fields. We only consider the gauge group G to be a classical Lie group since we wish to take a large- N_c limit so that the first inequality in (2.12) will be satisfied. We discuss two sets of theories, first those in which the gauge coupling is an exactly marginal operator and secondly models defined as isolated SCFTs. The gauge theories under consideration have either $\mathcal{N} = 2$ or $\mathcal{N} = 1$ supersymmetry in four dimensions; some of them have a known string theory dual while others do not (at this stage). Quite surprisingly, we find that in all of these models $\delta \geq 0$, which would seem to indicate a violation of the KSS bound (1.1). However, generically, $\delta \sim 1$ as $N_c \rightarrow \infty$ and so the second inequality in (2.12) is not satisfied. Therefore those theories do not have a gravity dual with a controllable derivative expansion, which is required for (2.13) to be valid. A similar analysis was carried out by Yuji Tachikawa and Brian Wecht [44]. Recently, [45] presented a complementary analysis of super-QCD with various relevant superpotentials. Related calculations also appear in [46].

A superconformal gauge theory has an anomaly free global $U(1)_R$ symmetry. The central charges, a and c , are relatively easy to determine as they are related to gravitational

anomalies in this global symmetry. Consider a superconformal gauge theory with a gauge group G and matter multiplets in representations $\{R_i\}$. Let r_i denote the R -charges of the matter chiral multiplets⁵ in the representation R_i . It was found in [47] that

$$\begin{aligned}
 c - a &= -\frac{1}{16} \left(\dim G + \sum_i (\dim R_i) (r_i - 1) \right), \\
 c &= \frac{1}{32} \left(4(\dim G) + \sum_i (\dim R_i) (1 - r_i) (5 - 9(1 - r_i)^2) \right).
 \end{aligned}
 \tag{3.2}$$

Thus computation of δ reduces to the identification of the anomaly-free $U(1)_R$ symmetry of the gauge theory at a superconformal fixed point. Our approach to this question depends whether the gauge coupling is marginal or the theory is at an isolated fixed point.

3.1 Superconformal gauge theories with exactly marginal gauge coupling

Let us begin with the identification of the anomaly-free $U(1)_R$ symmetry for the case where the gauge coupling is exactly marginal. Resolving this question is straightforward in this case as it can be shown a $U(1)_R$ symmetry with classical assignment of the R -charges is anomaly free, given the vanishing of the one-loop perturbative β -function. Consider classical assignment of R -charges, i.e., all matter superfields have $r_i = \frac{2}{3}$ and a vector superfield has $r_{\text{adj}} = 1$. The superconformal algebra then implies that anomalous dimensions of chiral superfields (χsf) must vanish. That is, the vanishing of the NSVZ exact perturbative β -function, which is equivalent for zero anomalous dimensions to vanishing of one-loop perturbative β -function,

$$0 = \beta_{\mathcal{N}=1}(g) \propto \left(\frac{3}{2} T(\text{adj}) - \frac{1}{2} \sum_{i \in \chi sf} T(R_i) \right),
 \tag{3.3}$$

guarantees that the classical R -charge assignment is in fact anomaly-free:

$$\begin{aligned}
 \langle \partial_\mu j_R^\mu \rangle &\propto \left(r_{\text{adj}} T(\text{adj}) + \sum_{i \in \chi sf} (r_i - 1) T(R_i) \right) = \left(T(\text{adj}) - \frac{1}{3} \sum_{i \in \chi sf} T(R_i) \right) \\
 &\propto \beta_{\mathcal{N}=1}(g) = 0.
 \end{aligned}
 \tag{3.4}$$

In these expressions, $T(\text{adj})$ and $T(R_i)$ are group indices of the adjoint representation and a χsf representation R_i in G , e.g., see [48] for explicit values. We only consider non-chiral theories in the following.

3.1.1 $SU(N_c)$

Since $T(\text{adj}) = 2N_c$, to satisfy the vanishing of β -function as $N_c \rightarrow \infty$, we can consider (besides adjoint) only fundamental, symmetric and antisymmetric representations for the χsf — any other representation has an index growing at least as $\mathcal{O}(N_c^2)$ as $N_c \rightarrow \infty$.

⁵We use $\mathcal{N} = 1$ susy representations to describe theories with extended supersymmetry as well.

Suppose we have n_{adj} χsf in the adjoint representation, n_f flavors⁶ in the fundamental representation, n_{sym} flavors in the symmetric representation and n_{asym} flavors in the anti-symmetric representation. Then, the vanishing of the NSVZ β -function implies

$$0 = \frac{3}{2} \cdot 2N_c - \frac{1}{2} \left(n_{\text{adj}} \cdot 2N_c + 2n_f \cdot 1 + 2n_{\text{sym}} \cdot (N_c + 2) + 2n_{\text{asym}} \cdot (N_c - 2) \right), \quad (3.5)$$

which we can rearrange to yield

$$n_f = (3 - n_{\text{adj}} - n_{\text{sym}} - n_{\text{asym}})N_c + 2(n_{\text{asym}} - n_{\text{sym}}). \quad (3.6)$$

Using this result, we can rewrite $(c - a)$ in (3.2) as

$$c - a = \frac{N_c^2}{16} \left(1 - \frac{1}{3}(n_{\text{adj}} + n_{\text{sym}} + n_{\text{asym}}) \right) + \frac{N_c}{16} (n_{\text{asym}} - n_{\text{sym}}) + \frac{1}{16} \left(1 - \frac{1}{3}n_{\text{adj}} \right). \quad (3.7)$$

Since $c \sim \mathcal{O}(N_c^2)$, requiring that $\delta \ll 1$ as $N_c \rightarrow \infty$ necessitates

$$n_{\text{adj}} + n_{\text{sym}} + n_{\text{asym}} = 3, \quad (3.8)$$

which along with $n_f \geq 0$ further implies that

$$n_{\text{asym}} - n_{\text{sym}} \geq 0. \quad (3.9)$$

It is easy now to enumerate all the models with $G = \text{SU}(N_c)$ and $\delta \ll 1$ as $N_c \rightarrow \infty$, as shown in the following table.

	$(n_{\text{adj}}, n_{\text{asym}}, n_{\text{sym}}, n_f)$	$c - a$	δ
(a)	(3,0,0,0)	0	0
(b)	(2,1,0,1)	$\frac{3N_c+1}{48}$	$\frac{1}{4N_c} + \mathcal{O}(N_c^{-2})$
(c)	(1,2,0,2)	$\frac{3N_c+1}{24}$	$\frac{1}{2N_c} + \mathcal{O}(N_c^{-2})$
(d)	(1,1,1,0)	$\frac{1}{24}$	$\frac{1}{6N_c^2} + \mathcal{O}(N_c^{-4})$
(e)	(0,3,0,3)	$\frac{3N_c+1}{16}$	$\frac{3}{4N_c} + \mathcal{O}(N_c^{-2})$
(f)	(0,2,1,1)	$\frac{N_c+1}{16}$	$\frac{1}{4N_c} + \mathcal{O}(N_c^{-2})$

Notice that model (a) has a matter content corresponding to $\mathcal{N} = 4$ susy (and as a result $\delta_{(a)} = 0$). Similarly, models (c) and (d) have a matter content corresponding to $\mathcal{N} = 2$ susy. In principle, the five models (b) through (f) are described by a gravity dual with the effective action (2.3). Further, we note that $\delta > 0$ for each of these models and so they would seem to give violations of the KSS bound (1.1). However, the gauge coupling is marginal in all of these models and so we would have to make sure there is a regime in which δ gives the dominant correction to the ratio of the shear-viscosity-to-entropy-density (2.13). As discussed at the end of section 2, if we imagine that the gravity dual comes from a string theory construction, this should be possible for models (b, c, e, f) with $\delta \sim 1/N_c$ if the inequality (2.27) is satisfied. However, for model (d) with $\delta \sim 1/N_c^2$, we should note that the R^4 interactions are also expected to contribute (positive) corrections at $\mathcal{O}(\lambda^{1/2}/N_c^2)$. The latter would always dominate since $\lambda \gg 1$ is also required for a sensible derivative expansion. However, the four superconformal gauge theories (b, c, e, f) potentially have string theory duals which would produce violations of the KSS bound.

⁶Recall that for a chiral representation, one flavor is the sum of two conjugate representations.

3.1.2 $SO(2N_c + 1)$ and $SO(2N_c)$

The analysis proceeds precisely as before. As well as $n_{\text{adj}} \chi sf$ in the adjoint (or antisymmetric) representation, we consider $n_v \chi sf$ in the vector representation and $n_{\text{sym}} \chi sf$ in the symmetric representation. For general $(n_{\text{adj}}, n_{\text{sym}}, n_v)$, subject to vanishing β -function, we find as $N_c \rightarrow \infty$

$$\delta = \frac{1}{2} \frac{3 - n_{\text{adj}} - n_{\text{sym}}}{9 - n_{\text{adj}} - n_{\text{sym}}} + \mathcal{O}(N_c^{-1}), \tag{3.10}$$

which is supplemented with the condition

$$n_v \geq 0, \quad \implies \quad 3 - n_{\text{adj}} - n_{\text{sym}} \geq 0. \tag{3.11}$$

This suggests that while $\delta \geq 0$, the only model with a controllable gravitational dual is the one with $\delta = 0$ and the $\mathcal{N} = 4$ susy matter content since when $n_{\text{adj}} + n_{\text{sym}} = 3$, the condition $n_v \geq 0$ also requires that $n_{\text{sym}} = 0$.

3.1.3 $Sp(N_c)$

The analysis is the same as before. Besides $n_{\text{adj}} \chi sf$ in the adjoint (symmetric) representation, we consider $n_f \chi sf$ in the fundamental representation and $n_{\text{asym}} \chi sf$ in the antisymmetric representation.

It is straightforward to establish that $\delta \geq 0$ always as $N_c \rightarrow \infty$ and to enumerate all the models with $\delta \ll 1$:

	$(n_{\text{adj}}, n_{\text{asym}}, n_f)$	$c - a$	δ
(a)	(3,0,0)	0	0
(b)	(2,1,4)	$\frac{6N_c - 1}{48}$	$\frac{1}{4N_c} + \mathcal{O}(N_c^{-2})$
(c)	(1,2,8)	$\frac{6N_c - 1}{24}$	$\frac{1}{2N_c} + \mathcal{O}(N_c^{-2})$
(d)	(0,3,12)	$\frac{6N_c - 1}{16}$	$\frac{3}{4N_c} + \mathcal{O}(N_c^{-2})$

Notice that model (a) has a matter content corresponding to $\mathcal{N} = 4$ susy (and as a result $\delta_{(a)} = 0$). Model (c) is that originally identified by Kats and Petrov [24] and has a matter content corresponding to $\mathcal{N} = 2$ susy. Models (b) and (d) provide interesting new candidates for a controllable gravity dual which again yield violations of the KSS bound (1.1).

3.2 Isolated superconformal fixed points

There are several ways to engineer an isolated superconformal fixed point. In a purely field theoretical construction, we can define an asymptotically free gauge theory in the UV, which flows to a strongly coupled interactive conformal fixed point in the IR [49, 50]. These models have $\mathcal{N} = 1$ supersymmetry. Alternatively, one can engineer isolated superconformal fixed points arising from the large number of D3-branes at singularities in F-theory [51–55]. The latter have $\mathcal{N} = 2$ supersymmetry. All these theories have non-classical assignment of R -charges of the anomaly-free global $U(1)_R$ symmetry for matter fields, which implies $\mathcal{O}(1)$ anomalous dimensions of chiral superfields — of course, this is simply a reflection of the strong coupling at the isolated superconformal fixed point. Unlike the examples

of superconformal fixed points with exactly marginal coupling discussed above, in the models which we review here with $\mathcal{N} = 1$ supersymmetry, δ is always positive but it is not suppressed by inverse powers of N_c in the large N_c limit. Therefore these theories will not have a controllable gravity dual and cannot be proven with the approach considered here to give counterexamples to the KSS bound.⁷ On the other hand, the models of [42] engineered directly in string theory can violate the KSS bound (1.1).

3.2.1 Conformal window for $\mathcal{N} = 1$ $SU(N_c)$ gauge theory

Consider $\mathcal{N} = 1$ $SU(N_c)$ gauge theory with n_f flavors in the fundamental representation. As shown in [49], for $\frac{3}{2}N_c < n_f < 3N_c$ the theory flows to a nontrivial superconformal fixed point in the IR. The matter fields global anomaly-free $U(1)_R$ charge assignment is as follows [49]:

$$r_i = 1 - \frac{N_c}{n_f}, \tag{3.12}$$

which from (3.2) implies

$$c - a = \frac{N_c^2 + 1}{16}, \quad \delta = \frac{n_f^2}{7n_f^2 - 9N_c^2} + \mathcal{O}(N_c^{-2}, n_f^{-2}). \tag{3.13}$$

The work of [45] expands on these results by adding adjoint matter fields and studying the effect of various superpotential terms. In these theories, they again find that δ is always positive but also order one in the limit of large N_c .

3.2.2 Conformal window for $\mathcal{N} = 1$ $SO(N_c)$ gauge theory

Consider $\mathcal{N} = 1$ $SO(N_c)$ gauge theory with n_f flavors in the vector representation. As shown in [49], for $\frac{3}{2}(N_c - 2) < n_f < 3(N_c - 2)$ the theory flows to a nontrivial superconformal fixed point in the IR. The matter fields global anomaly-free $U(1)_R$ charge assignment is as follows [49]:

$$r_i = 1 - \frac{N_c - 2}{n_f}, \tag{3.14}$$

which from (3.2) implies

$$c - a = \frac{N_c(N_c - 3)}{32}, \quad \delta = \frac{n_f^2}{7n_f^2 - 9N_c^2} + \mathcal{O}(N_c^{-1}). \tag{3.15}$$

3.2.3 Conformal window for $\mathcal{N} = 1$ $Sp(N_c)$ gauge theory

Consider $\mathcal{N} = 1$ $Sp(N_c)$ gauge theory with $2n_f$ flavors in the fundamental representation. As shown in [50], for $\frac{3}{2}(N_c + 1) < n_f < 3(N_c + 1)$ the theory flows to a nontrivial superconformal fixed point in the IR. The matter fields global anomaly-free $U(1)_R$ charge assignment is as follows [50]:

$$r_i = 1 - \frac{N_c + 1}{n_f}, \tag{3.16}$$

⁷Besides models listed below, we have considered Kutasov-Schwimmer model [56] and there again we find $\delta > 0$ but $\delta \sim 1$.

which from (3.2) implies

$$c - a = \frac{N_c(2N_c + 3)}{16}, \quad \delta = \frac{n_f^2}{7n_f^2 - 9N_c^2} + \mathcal{O}(N_c^{-1}). \quad (3.17)$$

3.2.4 $\mathcal{N} = 2$ superconformal fixed points from F-theory

Models constructed as N_c D3-branes probing an F-theory singularity generated by a collection of n_7 coincident (p, q) 7-branes and resulting in a constant dilaton were classified in [42]. Classifying the F-theory singularity with the symmetry group \mathcal{G} , one finds [42]

\mathcal{G}	H_0	H_1	H_2	D_4	E_6	E_7	E_8
n_7	2	3	4	6	8	9	10

We emphasize that the F-theory analysis fully accounts for the back-reaction of the n_7 7-branes, which generate a deficit angle $\pi n_7/6$ in the internal geometry. Central charges of the dual four-dimensional $\mathcal{N} = 2$ superconformal gauge theories were also computed [42] and as $N_c \rightarrow \infty$, one has

$$c - a = \frac{1}{4} \left(\frac{n_7}{12 - n_7} \right) N_c - \frac{1}{24}, \quad \delta = \frac{n_7}{12} \frac{1}{N_c} + \mathcal{O}(N_c^{-2}). \quad (3.18)$$

Notice then that with large but finite N_c , each of the models tabulated above yields $\delta > 0$ and $\delta \ll 1$. Hence they all have a controllable gravity dual and (2.13) yields a violation of the KSS bound (1.1). The string coupling remains arbitrary in the D_4 model and so this actually corresponds to a superconformal gauge theory with an exactly marginal operator. Of course, this is precisely the case that was examined by Kats and Petrov [24].⁸

4 The strongly coupled quark-gluon plasma?

Our analysis in section 2 demonstrates that the thermal properties of a large class of conformal gauge theories can be derived from a simple holographic framework. Of course, one is tempted to consider how these results might be applied to understand the strongly coupled quark-gluon plasma, which is currently under study with experiments at RHIC and soon at the LHC. In this direction, we would like to generalize a phenomenological approach originally advocated in [20]. The essential first step is to assume that the QCD plasma is described by an effective conformal field theory. Given this assumption, this effective CFT will have a holographic dual according to the AdS/CFT correspondence and if nature is gracious, the dual theory may be one for which we calculate. That is, the holographic dual may be approximated by the five-dimensional Einstein gravity coupled to a negative cosmological constant, with controllable higher curvature corrections.

In this case, we can ask if the sQGP is described by an effective CFT within the class of theories whose dual is governed the low energy action (2.3). We can then treat the parameters characterizing the CFT, i.e., the central charges a and c , or equivalently the dual

⁸In the present F-theory description, the O7-plane is resolved as a combination of a (-1,-1) and a (1,-3) 7-brane.

gravitational parameters ℓ_P/L and α_3 , as phenomenological. That is, we can calculate the properties of the gauge theory plasma from the gravity dual and then compare the results to experimental observations of QCD to fix the effective parameters. One interesting property for such a comparison would be η/s , as given in (2.13). As discussed in [20], if we denote the energy density of the conformal plasma and of the corresponding free theory as ε and ε_0 , then the ratio $\varepsilon/\varepsilon_0$ provides another interesting quantity for comparison, as the ratio can also be determined by lattice QCD calculations. Hence our next step is to determine $\varepsilon/\varepsilon_0$ holographically with the effective action (2.3).

Working to first order in α_3 or ℓ_P^2/L^2 , the equilibrium state of CFT plasma is encoded in the AdS₅-Schwarzschild background geometry [24]

$$ds_T^2 = \frac{r^2}{L^2} (-f(r)dt^2 + d\vec{x}^2) + \frac{L^2}{r^2} \frac{dr^2}{f(r)}, \tag{4.1}$$

where

$$f(r) = 1 - \frac{r_0^4}{r^4} + \frac{2}{3}\alpha_3 + 2\alpha_3 \frac{r_0^8}{r^8}. \tag{4.2}$$

The horizon appears at

$$r_H = r_0 \left(1 - \frac{2}{3}\alpha_3\right), \tag{4.3}$$

and the plasma temperature corresponds to the Hawking temperature which is given by

$$T = \frac{r_0}{\pi L^2} \left(1 - \frac{7}{3}\alpha_3\right). \tag{4.4}$$

Note that in order to obtain the temperature, we have to rescale the time coordinate to ensure that the speed of light in the CFT is unity. Next we evaluate the black hole entropy for the solution (4.1) following the standard approach of [57] for gravity actions with higher curvature corrections. The general expression takes the form

$$S = -2\pi \oint \frac{\delta \mathcal{L}}{\delta R_{abcd}} \hat{\varepsilon}_{ab} \hat{\varepsilon}_{cd} \bar{\varepsilon}. \tag{4.5}$$

Of course, in the present case with a planar horizon, the horizon area diverges and so we calculate the entropy density. Dividing by the coordinate volume, the final result can be expressed as

$$s = \frac{S}{V_{\text{CFT}}} = 2\pi \frac{L^3}{\ell_P^3} \left(\frac{r_H}{L^2}\right)^3 [1 + 4L^2 \alpha_3 R^{tr}_{tr}]_{r=r_H} = 2\pi \frac{L^3}{\ell_P^3} \left(\frac{r_H}{L^2}\right)^3 (1 + 8\alpha_3). \tag{4.6}$$

Note here that since the curvature above is multiplied by α_3 , we can evaluate it on the leading order solution, i.e., $R^{tr}_{tr}|_{r=r_0} = 2/L^2$. To express this result in terms of CFT parameters, we use the relations (2.11) as well as our expressions above for the horizon radius (4.3) and temperature (4.4). Combining all of these, we arrive at the final result

$$s \simeq 2\pi^2 c T^3 \left(1 + \frac{5}{4} \frac{c-a}{c}\right). \tag{4.7}$$

We would like to compare this result for the entropy density which is implicitly calculated for strong coupling to the entropy density of the free field limit. To produce a quantitative result, it turns out that we must assume that the underlying CFT is supersymmetric. We begin by noting that the central charges may be written as [27]:

$$\begin{aligned} a &= \frac{124 N_1 + 11 N_{1/2} + 2 N_0}{720}, \\ c &= \frac{12 N_1 + 3 N_{1/2} + N_0}{120}, \end{aligned} \tag{4.8}$$

where N_1 , $N_{1/2}$ and N_0 denote the number of vectors, (chiral) fermions and scalars, respectively. While these expressions assume that these are all massless free fields, the results are protected in a supersymmetric theory and so also apply at finite coupling in that case. In a supersymmetric theory, we have an equal number of bosonic and fermionic degrees of freedom, which we denote as $N = 2N_1 + N_0 = 2N_{1/2}$, and therefore the entropy density is naturally proportional to N . Hence we find the linear combination⁹

$$(2c - a) = \frac{2(2N_1 + N_0) + 5N_{1/2}}{144} = \frac{1}{32}N. \tag{4.9}$$

Now assuming we have a collection of free fields, the entropy density is easily calculated to be [58]

$$s_0 = \frac{\pi^2}{12}NT^3 = \frac{8\pi^2}{3}cT^3 \left(1 + \frac{c-a}{c}\right). \tag{4.10}$$

Hence comparing with (4.7), the ratio becomes

$$\frac{s}{s_0} = \frac{3}{4} \left(1 + \frac{1}{4} \frac{c-a}{c}\right). \tag{4.11}$$

Note that we recover the celebrated result $s/s_0 = 3/4$ with $c = a$ [58], in which case the gravity dual reduces to Einstein gravity (coupled to a negative cosmological constant). Further, however, the sign of the correction to the ratio here is the opposite to that for the ratio of shear viscosity to entropy density (2.13). Of course for a conformal (or free) field theory, the energy density and entropy density are simply related as $\varepsilon = \frac{4}{3}sT$. Hence the result in (4.11) applies equally well for the ratio of the energy densities of the strongly coupled and free theories.

Collecting our results then, all of the CFT's for which (2.3) represents the gravity dual will have the following:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{3}{4} \left(1 + \frac{1}{4} \delta\right) \quad \text{and} \quad \frac{\eta}{s} = \frac{1}{4\pi} (1 - \delta), \tag{4.12}$$

$$\text{where} \quad \delta \equiv \frac{c-a}{c}. \tag{4.13}$$

In principle, the CFT's in the class of interest here have two independent parameters, a and c , but above we have chosen two quantities which only depend on the combination

⁹The same result follows from adding together the two expressions in (3.2) and evaluating the result with all $r_i = 2/3$.

δ . Thus we may treat δ as a phenomenological parameter under the assumption that the effective CFT describing the QCD plasma lies within this class. This assumption is then put to the test if both $\varepsilon/\varepsilon_0$ and η/s can be constrained by observation and consistently fit with the same value of δ . We can begin by using lattice QCD results to fix δ with the energy density. Recent studies seem to indicate that energy density should be in the range $\varepsilon/\varepsilon_0 \approx 0.85 - 0.90$ [59]. In this case, (4.12) yields $\delta \approx 0.53 - 0.80$ and hence

$$\left. \frac{\eta}{s} \right|_{\text{QCD}} \approx 0.016 - 0.037 . \tag{4.14}$$

These ‘corrected’ values for η/s are significantly lower than the leading result, i.e., the conjectured KSS bound $\eta/s|_{\text{KSS}} = 1/4\pi \simeq 0.08$ [7]. Even though the ‘correction’ to $\varepsilon/\varepsilon_0$ is small, our fit produced a range of large values for the parameter δ . In fact, these values are all too large since consistency of the effective CFT demands that $|\delta| < .5$ [31]. Hence we can conclude that the class of holographic models considered here cannot describe an effective CFT for the QCD plasma and we must broaden the universality class under consideration.

When considering higher order corrections, it is natural to take into account higher curvature terms in the effective gravity theory beyond the curvature-squared term appearing in (2.3). Naturally the next term to consider would involve a contraction of three Riemann tensors. The corresponding coupling constant would be dual to a new CFT parameter in the three point function of the stress tensor. However, one can argue that this parameter, and hence the dual gravitational coupling, vanishes for any supersymmetric CFT [31]. Since supersymmetry was an underlying assumption in the analysis above, it is natural then to set the R^3 term to zero.

As already discussed in section 2, string theory provides a specific interaction quartic in curvatures [41]. As considered there, in situations where the string coupling is a free parameter, this term does not necessarily enter the action suppressed by ℓ_p^6/L^6 . Hence the contributions of this R^4 term can be enhanced in certain regimes of the parameter space. For example in $\mathcal{N} = 4$ super-Yang-Mills, the correction to η/s is $15\zeta(3)/\lambda^{3/2}$ [19] and if we evaluate this contribution with $\lambda = 6\pi$, as might be applicable for the QCD plasma, it could easily compete with $1/N_c$ contributions (with $N_c = 3$ as in QCD). With this observation, we argue that it is not unreasonable to include the corrections from both the R^2 and R^4 terms as making independent and comparable contributions to the CFT properties, i.e.,

$$\frac{\varepsilon}{\varepsilon_0} = \frac{3}{4} \left(1 + \frac{1}{8}\Delta + \frac{1}{4}\delta \right) \quad \text{and} \quad \frac{\eta}{s} = \frac{1}{4\pi} (1 + \Delta - \delta) , \tag{4.15}$$

where Δ encodes the R^4 corrections [20]. Within the context of the phenomenological program advocated above, we have expanded the class of CFT’s which might describe the sQGP and so now have greater freedom in fitting the observed values of these quantities. In order to arrive at a constrained or predictive system, we have to calculate more physical properties of the QCD plasma. This does not present a real obstacle for the holographic framework since the effective gravity action allows us to calculate the corrections for any properties having to do with the stress-energy tensor. So in particular, we can calculate

corrections to the higher order transport coefficients [60] and with these we may be able to produce a constrained set of observables.

5 Discussion

We examined the effective low-energy description of the gauge/gravity duality, relevant for discussing thermal and hydrodynamic properties of strongly coupled conformal gauge theory plasmas. We argued that as long as the central charges of the CFT satisfy

$$c \sim a \gg 1 \quad \text{and} \quad |c - a|/c \ll 1, \quad (5.1)$$

the dual gravity description should be described with Einstein gravity coupled to a negative cosmological constant with perturbative corrections coming from a curvature-squared interaction. The standard results for the holographic conformal anomaly [28–30] precisely fix the relevant gravitational couplings in terms of the central charges. Our arguments assumed the validity of the effective field theory description in gravity dual, i.e., a reasonable derivative expansion and generic couplings for any matter fields. These assumptions may only be satisfied in a particular regime in theories with exactly marginal operators. In appendix B, we use type IIB string theory, more specifically type IIB supergravity plus probe Dp -branes (including leading ℓ_s^2 corrections) to establish that under certain conditions, holographic dualities can indeed be cast in the framework of the proposed low-energy description.

A primary motivation of our work was to examine the claim by Kats and Petrov [24] that the KSS bound (1.1) is violated in a certain string theory model. Our detailed analysis agrees that their calculations are in fact reliable and the bound is violated in the regime where $\lambda \gg N_c^{2/3}$, as they already noted. It is interesting that this restriction establishes the CFT coupling cannot be arbitrarily small if the KSS bound is to be violated. This is in keeping with the intuition that bound must not be violated at weak coupling because the viscosity grows arbitrarily large in the perturbative regime. This restriction can also be translated into a limit on how small the string coupling can be if the bound is violated in this string theory model, i.e., $g_s \gg N_c^{-1/3}$.

Often one also restricts the string coupling $g_s \ll 1$ to be in a perturbative regime where the microscopic details of the duality can be well understood, e.g., in our schematic discussion in appendix B. However, one important observation in section 2 is that these microscopic details are inessential to the low energy gravity action (2.3). Rather we can use the central charges to precisely fix the gravitational couplings with (2.11). Hence, as long as we can evaluate the central charges from the CFT and they satisfy the inequalities (5.1), we can reliably calculate the leading order corrections in δ with the effective action (2.3), irrespective of the string coupling. Hence the result (2.13) for η/s is still dependable for the F-theory models of [42] where the string coupling is fixed to be order one. As discussed in section 3, in the limit of large N_c , these models provide new examples where the KSS bound is violated.

In section 3, we also found various new superconformal gauge theories with $0 < \delta \sim 1/N_c$ (as well as $c \gg 1$) in the limit of large N_c . Even though no string theory model has (yet) been constructed which is dual to these gauge theories, by the arguments of section 2,

these CFT's will have a controllable gravity dual described by the action (2.3). Hence we can be confident that they also represent new examples where the KSS bound is violated. A caveat in these cases is that the gauge coupling is precisely marginal and so we expect that the bound will only be violated in the regime of large 't Hooft coupling. One's experience with the universal contributions of the R^4 interaction arising in string theory [20] suggests that we must require $\lambda \gg N_c^{2/3}$, as discussed for the example of [24]. In section 3, we also found one example where $0 < \delta \sim 1/N_c^2$ but we argued that the theory respects the KSS bound (in the large N_c limit) since λ cannot be tuned to a regime of where the R^2 contribution dominates.

A general feature that we found for all of the superconformal gauge theories analyzed in section 3 was that δ is positive. While we focussed there on cases with $\delta \ll 1$, all of our examples of $\mathcal{N} = 1$ theories which flowed to a nontrivial superconformal fixed point had δ was positive but $\delta \sim 1$ for N_c large — the same result applies for the examples in [45]. This feature is also the generic behaviour of the superconformal theories with an exactly marginal gauge coupling. For example, if we do not insist that $|\delta| \ll 1$, then we see that (3.7) still always yields $c - a > 0$ for large N_c because of the constraint that $N_f > 0$ combined with (3.6). However, this generic case yields $c - a \sim N_c^2$ and so $\delta \sim 1$. Although the gravity dual for such a CFT may be weakly curved, it would seem not to have a controlled derivative expansion. Therefore while we have found that $\delta > 0$ is generically positive for superconformal gauge theories, the implications of this observation remain unclear. Further, we should add that $\delta < 0$ can be achieved with a theory of free vector multiplets with $\mathcal{N} = 0, 1, 2$ supersymmetry [31, 32]. Of course, since these examples are free theories, they will not have a weakly curved gravity dual.

Our present discussion is limited to considering $|\delta| \ll 1$ and so any of our counterexamples to the KSS bound only produce small violations of the bound. However, this was simply a technical limitation arising since we need $|\delta| \ll 1$ to reliably formulate the gravity dual using the techniques of effective field theory. One might imagine that violations of the KSS bound still arise when $\delta \sim 1$, which as described above is the generic case, and further that these violations may become arbitrarily large in this case. However, on general grounds [8], one expects that η/s must remain finite and order one (in units where $\hbar = 1 = c = k_B$). Hence it is interesting then that basic considerations of three-point functions in any four-dimensional supersymmetric CFT seem to restrict $\delta \leq 1/2$ [31]. Further precisely the same bound was found by demanding causality in a holographic framework where the gravity dual incorporated the Gauss-Bonnet term as the curvature-squared interaction [23]. Taken at face value, the latter calculations suggest that the violations of the KSS bound are limited with $\eta/s \geq 16/100\pi$ for the superconformal gauge theories. However, firmly establishing a clear lower bound for η/s remains an open question.

In section 4, we advocated a phenomenological approach to applying the AdS/CFT correspondence to understanding the strongly coupled quark-gluon plasma of QCD. Assuming the sQGP is described by an effective conformal field theory the latter should be characterized by a few parameters controlling the aggregate properties of the plasma. With the AdS/CFT correspondence, these parameters would then fix the couplings of the dual gravity theory, e.g., as in (2.11). These parameters could then be fixed by comparing the

results determined by holographic calculations with those emerging from analysis of experimental data, as well as lattice calculations. By taking into account sufficiently many quantities, the comparison is constrained and one can concretely test the assumption that the QCD plasma is described by a CFT within the universality class defined by a certain family of gravity duals. With the gravity dual, we are restricting our attention to the properties of the CFT probed by the stress tensor, however, the holographic framework allows us to calculate any quantities originating with this operator. Hence, in principle, there is no problem in expanding the calculations to a sufficiently broad set of quantities so that the suggested comparison becomes constrained. At present, the obstruction to this phenomenological program is that the experimental data does not yet yield precision results for most quantities of interest.

Implicit in this discussion is also the assumption that the effective CFT describing the sQGP is close to Einstein gravity. That is, the gravity dual is Einstein gravity coupled to a negative cosmological constant with perturbative corrections coming from a limited number of higher curvature interactions. Again, this is simply a technical issue as our present understanding limits our holographic calculations to producing reliable results within this framework. The primary motivation to believe that nature could be so kind as to respect these limitations was that the value for the shear viscosity emerging from the RHIC data [6] is unusually small and even seems to be roughly $1/4\pi$, the universal result for Einstein gravity duals [8–17]. The present discussion may call this motivation into question. Above we found that the value of η/s can become smaller than $1/4\pi$ but suggested that it will not become too much smaller even if we go well beyond the regime where corrections to Einstein gravity can be treated perturbatively. Further, having realized that the KSS bound can be violated, we observe that if (4.15) is representative then $\eta/s = 1/4\pi$ only defines a codimension-one surface in the space of possible CFT's and so even if this precise value is found for the sQGP, it is not clear how close the effective CFT will be to having an Einstein gravity dual.

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A Comments on field redefinitions

As remarked in section 2, in general, the full CFT will have a spectrum of interesting operators, each of which will be dual to an independent field in the gravity theory. These

fields will appear in interactions at all orders in the derivative expansion and it is interesting to examine how field redefinitions can modify the higher derivative terms for such fields. For simplicity, we begin our discussion here by adding a single scalar field to the gravity dual. However, we will comment on the case of multiple scalars and other generalizations below. The most general four-derivative action for gravity coupled to a scalar field ϕ (as well as a negative cosmological constant) is:

$$\begin{aligned}
 I = \frac{1}{2\ell_p^3} \int d^5x \sqrt{-g} [& \mathcal{U}(\phi) + R - \mathcal{K}(\phi) \nabla\phi \cdot \nabla\phi \\
 & + \mathcal{A}_1(\phi) R^2 + \mathcal{A}_2(\phi) R_{ab}R^{ab} + \mathcal{A}_3(\phi) R_{abcd}R^{abcd} \\
 & + \mathcal{B}_1(\phi) \nabla\phi \cdot \nabla\phi R + \mathcal{B}_2(\phi) \square\phi R + \mathcal{B}_3(\phi) \nabla^a\phi \nabla^b\phi R_{ab} + \mathcal{B}_4(\phi) \nabla^a\nabla^b\phi R_{ab} \\
 & + \mathcal{C}_1(\phi) (\nabla\phi \cdot \nabla\phi)^2 + \mathcal{C}_2(\phi) (\square\phi)^2 + \mathcal{C}_3(\phi) \nabla\phi \cdot \nabla\phi \square\phi + \mathcal{C}_4(\phi) \square^2\phi \\
 & + \mathcal{C}_5(\phi) \nabla_a\phi \nabla^a \square\phi + \mathcal{C}_6(\phi) \nabla_a \nabla_b\phi \nabla^a \nabla^b\phi + \mathcal{C}_7(\phi) \nabla_a \nabla_b\phi \nabla^a \phi \nabla^b\phi] .
 \end{aligned} \tag{A.1}$$

In general, one might have expected an additional function $\mathcal{V}(\phi)$ to be multiplying the Einstein term, but implicitly we have eliminated such a coupling with a conformal transformation: $g_{ab} \rightarrow \mathcal{V}(\phi)^{-2/3} g_{ab}$. As in section 2, we have adopted the convention that ϕ has zero engineering dimension and we are also assuming that the various coefficient functions, e.g., \mathcal{A}_i , \mathcal{B}_i and \mathcal{C}_i , are nonsingular at $\phi = 0$. Many of the four-derivative terms above can be eliminated by simply integrating by parts. For example,

$$\begin{aligned}
 \int d^5x \sqrt{-g} \mathcal{C}_7(\phi) \nabla_a \nabla_b\phi \nabla^a \phi \nabla^b\phi & \\
 = -\frac{1}{2} \int d^5x \sqrt{-g} (\mathcal{C}'_7(\phi) (\nabla\phi \cdot \nabla\phi)^2 + \mathcal{C}_7(\phi) \nabla\phi \cdot \nabla\phi \square\phi) , &
 \end{aligned} \tag{A.2}$$

where $\mathcal{C}'_7 \equiv \delta\mathcal{C}_7/\delta\phi$. In this way, one can eliminate \mathcal{B}_4 , \mathcal{C}_4 , \mathcal{C}_5 , \mathcal{C}_6 and \mathcal{C}_7 . Hence the general four-derivative action can be reduced to

$$\begin{aligned}
 I = \frac{1}{2\ell_p^3} \int d^5x \sqrt{-g} [& \mathcal{U}(\phi) + R - \mathcal{K}(\phi) \nabla\phi \cdot \nabla\phi \\
 & + \mathcal{A}_1(\phi) R^2 + \mathcal{A}_2(\phi) R_{ab}R^{ab} + \mathcal{A}_3(\phi) R_{abcd}R^{abcd} \\
 & + \mathcal{B}_1(\phi) \nabla\phi \cdot \nabla\phi R + \mathcal{B}_2(\phi) \square\phi R + \mathcal{B}_3(\phi) \nabla^a\phi \nabla^b\phi R_{ab} \\
 & + \mathcal{C}_1(\phi) (\nabla\phi \cdot \nabla\phi)^2 + \mathcal{C}_2(\phi) (\square\phi)^2 + \mathcal{C}_3(\phi) \nabla\phi \cdot \nabla\phi \square\phi] .
 \end{aligned} \tag{A.3}$$

Now consider making field redefinitions: $g_{ab} \rightarrow g_{ab} + \delta g_{ab}$ and $\phi \rightarrow \phi + \delta\phi$. The most general field redefinition involving two-derivative contributions can be written

$$\begin{aligned}
 \delta g_{ab} = \mathcal{M}_1 R_{ab} + \mathcal{M}_2 \nabla_a \nabla_b\phi + \mathcal{M}_3 \nabla_a\phi \nabla_b\phi & \\
 + (\mathcal{M}_4 R + \mathcal{M}_5 \square\phi + \mathcal{M}_6 \nabla\phi \cdot \nabla\phi + \mathcal{M}_7) g_{ab} , & \\
 \delta\phi = \mathcal{N}_1 R + \mathcal{N}_2 \square\phi + \mathcal{N}_3 \nabla\phi \cdot \nabla\phi . &
 \end{aligned} \tag{A.4}$$

In these expressions, all of the \mathcal{M}_i and \mathcal{N}_i are understood to be functions of ϕ which are nonsingular at $\phi = 0$ and they are of order ℓ_p^2 . With these field redefinitions, the leading

change in the action is

$$\delta I = \frac{1}{2\ell_{\text{P}}^3} \int d^5x \sqrt{-g} \left\{ \left[\frac{1}{2} (\mathcal{U}(\phi) + R - \mathcal{K}(\phi) \nabla\phi \cdot \nabla\phi) g^{ab} - R^{ab} + \mathcal{K}(\phi) \nabla^a \phi \nabla^b \phi \right] \delta g_{ab} + \left(\mathcal{U}'(\phi) - 2\mathcal{K}(\phi) \square\phi - \mathcal{K}'(\phi) \nabla\phi \cdot \nabla\phi \right) \delta\phi \right\} \quad (\text{A.5})$$

$$\begin{aligned} = & \frac{1}{2\ell_{\text{P}}^3} \int d^5x \sqrt{-g} \frac{1}{2} \left\{ 5\mathcal{M}_7 \mathcal{U} + ((\mathcal{M}_1 + 5\mathcal{M}_4) \mathcal{U} + \mathcal{M}_7) R \right. \\ & + ((\mathcal{M}_3 + 5\mathcal{M}_6) \mathcal{U} + 3\mathcal{M}_7 \mathcal{K} - [(\mathcal{M}_2 + 5\mathcal{M}_5) \mathcal{U}]') (\nabla\phi)^2 \\ & + (\mathcal{M}_1 + 3\mathcal{M}_4) R^2 - \mathcal{M}_1 R_{ab} R^{ab} + (\mathcal{M}_2 + 3\mathcal{M}_5 - 4\mathcal{N}_1 \mathcal{K}) \square\phi R \\ & + (\mathcal{M}_3 + 3\mathcal{M}_6 + (\mathcal{M}_1 + 3\mathcal{M}_4) \mathcal{K}) (\nabla\phi)^2 R - 2(\mathcal{M}_2 - \mathcal{M}_1 \mathcal{K}) \nabla^a \phi \nabla^b \phi R_{ab} \\ & + [(\mathcal{M}_2 \mathcal{K}]' - (\mathcal{M}_3 - 3\mathcal{M}_6) \mathcal{K} - 2\mathcal{N}_3 \mathcal{K}') (\nabla\phi \cdot \nabla\phi)^2 - 4\mathcal{N}_2 \mathcal{K} (\square\phi)^2 \\ & \left. + ((2\mathcal{M}_2 + 3\mathcal{M}_5 - 4\mathcal{N}_3) \mathcal{K} - 2\mathcal{N}_2 \mathcal{K}') (\nabla\phi)^2 \square\phi \right\}, \quad (\text{A.6}) \end{aligned}$$

where as above, the prime indicates a derivative with respect to ϕ . Note that we have integrated by parts to produce the expressions in (A.6). Now given this result is should be clear that we have more than enough freedom to eliminate all of the four-derivative scalar terms in (A.3), i.e., we can set to zero the coefficients $\mathcal{A}_{1,2}$, $\mathcal{B}_{1,2,3}$ and $\mathcal{C}_{1,2,3}$. While we do not present the precise choices needed to produce these cancellations, we note the various couplings in (A.3) can be eliminated by fixing in turn various coefficients appearing in the field redefinitions (A.4), as follows: $(\mathcal{A}_1, \mathcal{M}_4)$, $(\mathcal{A}_2, \mathcal{M}_1)$, $(\mathcal{B}_1, \mathcal{M}_3)$, $(\mathcal{B}_2, \mathcal{N}_1)$, $(\mathcal{B}_3, \mathcal{M}_2)$, $(\mathcal{C}_1, \mathcal{N}_3)$, $(\mathcal{C}_2, \mathcal{N}_2)$, $(\mathcal{C}_3, \mathcal{M}_5)$. This leaves \mathcal{M}_6 and \mathcal{M}_7 undetermined. We can use the freedom in \mathcal{M}_7 to prevent any scalar couplings appearing in the Einstein term after the field redefinition and to keep the Planck scale fixed. Hence the field redefinitions (A.4), as well as integrating by parts, allow us to simplify the general action (A.1) down to

$$I = \frac{1}{2\ell_{\text{P}}^3} \int d^5x \sqrt{-g} \left[\mathcal{U}(\phi) + R - \mathcal{K}(\phi) \nabla\phi \cdot \nabla\phi + \mathcal{A}_3(\phi) R_{abcd} R^{abcd} \right]. \quad (\text{A.7})$$

Given this result, it is clear that none of the higher order terms involving derivatives of the scalar can be relevant in calculating quantities such as the shear viscosity.

Unfortunately, it turns out that field redefinitions are not as effective in eliminating four-derivative interactions when the effective theory involves many scalars ϕ^k . In this case, the coefficients of each of the scalar field interactions in (A.1) become “tensors” with indices to describe the various independent interactions involving different combinations of scalars. Hence the general four-derivative action for gravity coupled to a set of scalar fields

ϕ^k becomes:

$$\begin{aligned}
I = \frac{1}{2\ell_{\text{P}}^3} \int d^5x \sqrt{-g} & \left[\mathcal{U}(\phi^m) + R - \mathcal{K}_{ij}(\phi^m) \nabla\phi^i \cdot \nabla\phi^j \right. \\
& + \mathcal{A}_1(\phi^m) R^2 + \mathcal{A}_2(\phi^m) R_{ab} R^{ab} + \mathcal{A}_3(\phi^m) R_{abcd} R^{abcd} \\
& + \mathcal{B}_{1ij}(\phi^m) \nabla\phi^i \cdot \nabla\phi^j R + \mathcal{B}_{2i}(\phi^m) \square\phi^i R + \mathcal{B}_{3ij}(\phi^m) \nabla^a\phi^i \nabla^b\phi^j R_{ab} \\
& + \mathcal{B}_{4i}(\phi^m) \nabla^a \nabla^b \phi^i R_{ab} + \mathcal{C}_{1ijkl}(\phi^m) \nabla\phi^i \cdot \nabla\phi^j \nabla\phi^k \cdot \nabla\phi^l + \mathcal{C}_{2ij}(\phi^m) \square\phi^i \square\phi^j \\
& + \mathcal{C}_{3ijk}(\phi^m) \nabla\phi^i \cdot \nabla\phi^j \square\phi^k + \mathcal{C}_{4i}(\phi^m) \square^2\phi^i + \mathcal{C}_{5ij}(\phi^m) \nabla_a\phi^i \nabla^a \square\phi^j \\
& \left. + \mathcal{C}_{6ij}(\phi^m) \nabla_a \nabla_b \phi^i \nabla^a \nabla^b \phi^j + \mathcal{C}_{7ijk}(\phi^m) \nabla_a \nabla_b \phi^i \nabla^a \phi^j \nabla^b \phi^k \right].
\end{aligned} \tag{A.8}$$

Again, many of the four-derivative terms can be eliminated by simply integrating by parts. However, there is one complication in considering \mathcal{C}_{7ijk} . The natural extension of (A.2) now comes from considering the following total derivative:

$$\begin{aligned}
\nabla_a \left(\mathcal{C}_{7(ij)k}(\phi^m) \nabla\phi^i \cdot \nabla\phi^j \nabla^a \phi^k \right) & = 2\mathcal{C}_{7(ij)k}(\phi^m) \nabla_a \nabla_b \phi^i \nabla^b \phi^j \nabla^a \phi^k \\
& + \mathcal{C}_{7(ij)k}(\phi^m) \nabla\phi^i \cdot \nabla\phi^j \square\phi^k + \partial_l \mathcal{C}_{7(ij)k}(\phi^m) \nabla\phi^i \cdot \nabla\phi^j \nabla\phi^k \cdot \nabla\phi^l.
\end{aligned} \tag{A.9}$$

Above, the parentheses on the subscripts indicate symmetrization of the indices, i.e., $\mathcal{C}_{7(ij)k} = \frac{1}{2}(\mathcal{C}_{7ijk} + \mathcal{C}_{7jik})$. In general then, the coefficients \mathcal{C}_{7ijk} do not have to be symmetric in the indices i and j but because of the form of the tensor in the total derivative (A.9), integrating by parts can only eliminate the symmetric combination $\mathcal{C}_{7(ij)k}$. Hence the natural generalization of (A.3) is slightly more involved in the case of multiple scalars. First we must add indices as appropriate in the interactions appearing there but we must also include an extra term proportional to $\mathcal{C}_{7[ij]k} = \frac{1}{2}(\mathcal{C}_{7ijk} - \mathcal{C}_{7jik})$.

Next we wish to consider the field redefinitions generalizing those in (A.4), i.e., $g_{ab} \rightarrow g_{ab} + \delta g_{ab}$ and $\phi^i \rightarrow \phi^i + \delta\phi^i$ with

$$\begin{aligned}
\delta g_{ab} & = \mathcal{M}_1 R_{ab} + \mathcal{M}_{2i} \nabla_a \nabla_b \phi^i + \mathcal{M}_{3ij} \nabla_a \phi^i \nabla_b \phi^j \\
& + (\mathcal{M}_4 R + \mathcal{M}_{5i} \square\phi^i + \mathcal{M}_{6ij} \nabla\phi^i \cdot \nabla\phi^j + \mathcal{M}_7) g_{ab}, \\
\delta\phi^i & = \mathcal{N}_1^i R + \mathcal{N}_{2j}^i \square\phi^j + \mathcal{N}_{3jk}^i \nabla\phi^j \cdot \nabla\phi^k.
\end{aligned} \tag{A.10}$$

With these field redefinitions, we can consider the leading change in the action but this exercise is rather tedious and so we give only a schematic description of the results. In certain cases, the previous discussion follows through unchanged. For example above, we canceled \mathcal{B}_1 by fixing the coefficient \mathcal{M}_3 . Here this pairing becomes $(\mathcal{B}_{1(ij)}, \mathcal{M}_{3(ij)})$. The structure of the corresponding terms is such that both of these expressions are symmetric in their subscripts, as indicated by the parentheses. Hence the index or tensor properties match nicely in this particular case and it is clear that there are precisely enough degrees of freedom in $\mathcal{M}_{3(ij)}$ to eliminate $\mathcal{B}_{1(ij)}$. However, in a number of cases, there is a mismatch for the tensor expressions. For example, the pairing $(\mathcal{C}_3, \mathcal{M}_5)$ becomes with multiple scalars, $(\mathcal{C}_{3(ij)k}, \mathcal{M}_{5i})$. Hence in this case, it is clear that in general with more than one scalar field, there are not enough degrees of freedom in the field redefinition \mathcal{M}_{5i} to eliminate all of the possible couplings $\mathcal{C}_{3(ij)k}$. Our final result is that we still have the freedom to set to

zero the couplings, \mathcal{A}_1 , \mathcal{A}_2 , $\mathcal{B}_{1(ij)}$, \mathcal{B}_{2i} and $\mathcal{C}_{2(ij)}$, but we can only partially eliminate $\mathcal{B}_{3(ij)}$, $\mathcal{C}_{1(ij)(kl)}$ and $\mathcal{C}_{3(ij)k}$. Further as described above, a new set of couplings arise from $\mathcal{C}_{7[ij]k}$. Of course, in any given theory, it may be that the full set of general couplings does not appear, e.g., there might be internal symmetries which restrict the number and form of the independent couplings.

Hence after making the $\mathcal{O}(\ell_p^2)$ field redefinitions (A.10), the general four-derivative action (A.8) can be simplified to take the form

$$\begin{aligned}
 I = \frac{1}{2\ell_p^3} \int d^5x \sqrt{-g} & \left[\mathcal{U}(\phi^m) + R - \mathcal{K}_{ij}(\phi^k) \nabla\phi^i \cdot \nabla\phi^j + \mathcal{A}_3(\phi^m) R_{abcd} R^{abcd} \right. \\
 & + \mathcal{B}_{3(ij)}(\phi^m) \nabla^a\phi^i \nabla^b\phi^j R_{ab} + \mathcal{C}_{1(ij)(kl)}(\phi^m) \nabla\phi^i \cdot \nabla\phi^j \nabla\phi^k \cdot \nabla\phi^l \\
 & \left. + \mathcal{C}_{3(ij)k}(\phi^m) \nabla\phi^i \cdot \nabla\phi^j \square\phi^k + \mathcal{C}_{7[ij]k}(\phi^m) \nabla_a \nabla_b \phi^i \nabla^b \phi^j \nabla^a \phi^k \right]. \quad (\text{A.11})
 \end{aligned}$$

At this point, an important observation is that the higher order interactions in the second and third line of this action contain at least two factors with derivatives of the scalars. Hence, since we are treating these terms perturbatively, if the scalars are constant in the leading solution, they will remain constant at the next order. Certainly, the scalars are constant in the leading order background of an AdS₅ black hole (4.1) and so the scalars will not effect the thermodynamic properties of dual CFT (at least at this order in the expansion in $(c - a)/c$). That is, these new coefficients define new parameters of the CFT which characterize certain correlators of the new operators (dual to the scalars) and the stress tensor. However, the properties of the thermal stress tensor are independent of these parameters at this order.

This discussion can be further extended to include vectors in the gravity theory. While a complete discussion requires an even more elaborate analysis, it is relatively straightforward to show that any new four-derivative interactions are at least quadratic in the field strengths of the gauge fields, with one exception. Hence an argument similar to that above applies here as well, with the conclusion that these terms will not effect the CFT's thermal properties, at this order. The one exception to these statements is as follows: In five dimensions with a U(1) gauge field, we can introduce an interaction: $\int A \wedge R^a{}_b \wedge R^b{}_a$. This term plays an interesting role in describing the anomaly for the U(1)_R current [36, 37] — see also [38] for recent supergravity analysis of this term. Since this interaction is linear in the gauge potential, it will induce a nontrivial profile in a background where $R^a{}_b \wedge R^b{}_a$ is nonvanishing. However, this combination of curvatures vanishes both for the AdS₅ vacuum and an AdS₅ black hole background. Hence we can conclude again that this term will play no role in determining the thermal properties of the CFT.

B String theory origin of R^2

In the string theory example considered by Kats and Petrov [24] and more generally in the F-theory constructions of [42], the curvature-squared interaction is argued to arise from the world-volume action of the D7-branes [37, 42]. In this appendix, we would like to develop a schematic understanding of the parameter dependence of the coupling coefficients in these

higher derivative interactions. In particular, we contrast these couplings with the analogous coefficients in the celebrated R^4 interaction [41] that arises from the closed string sector. Hence we are able to confirm the conditions (2.27) under which the R^2 corrections coming from the branes dominate over the bulk corrections arising the R^4 interaction. Along the way we will motivate the usage of the R^2 terms in eq. (2.15). We must note though that the final results rely on treating the Dp-branes as probe branes and so the discussion has more limited applicability than the effective action approach in section 2. Schematically we can write

$$S = \kappa_1 \int d^{10}x \sqrt{-g} (R - F_5^2 + \alpha'^3 R^4 + \dots) - \kappa_2 \int d^{p+1}x (\sqrt{G + F} + \alpha'^2 \sqrt{G} R^2 + \dots). \quad (\text{B.1})$$

We are in Einstein frame. Terms arising at ℓ_s^6 in the bulk action are generically denoted by R^4 . By R^2 we mean a generic term arising at ℓ_s^4 order in the brane action. It is known from scattering amplitude calculations off D-branes and O-planes [62] that these terms arise as stringy corrections to the DBI action. Here

$$\kappa_1 \sim \frac{1}{g_s^2 \ell_s^8}, \quad \kappa_2 \sim \frac{N_f}{g_s \ell_s^{p+1}}. \quad (\text{B.2})$$

κ_1 is essentially the inverse of the ten-dimensional Newton's constant while κ_2 is related to the tension of the Dp-brane. We will consider $p > 3$ so that we can get a 5d action by wrapping the brane on some $p - 4$ cycle. So $p = 5, 7, 9$.

Now we know from the standard AdS/CFT dictionary that

$$\ell_s^2 \sim \frac{1}{\sqrt{\lambda}}, \quad g_s \sim \frac{\lambda}{N_c}, \quad (\text{B.3})$$

where for convenience we have set the AdS radius to unity. Thus when λ is large, the six and higher derivative terms in the full brane action can be ignored compared to the four-derivative terms. Then in terms of these variables

$$S \sim N_c^2 \left(\int d^{10}x \sqrt{-g} \left(R - F_5^2 + \frac{1}{\lambda^{3/2}} R^4 + \dots \right) - N_f \int d^{p+1}x \frac{1}{N_c \lambda^{(3-p)/4}} \sqrt{G + F} + \frac{1}{N_c \lambda^{(7-p)/4}} \sqrt{G} R^2 + \dots \right). \quad (\text{B.4})$$

Thus for $p = 7$, the first term in the DBI action leads to a correction to the effective cosmological constant of $\mathcal{O}(\lambda/N_c)$ while the second term gives an R^2 term with coefficient $\mathcal{O}(1/N_c)$. In order to produce a 5d theory, we need to integrate the brane action over some $(p-4)$ -cycle. We have to ensure that the volume of this $(p-4)$ -cycle satisfies $V_{p-4} \gg \ell_s^3$ for the derivative expansion to make sense. For general p , from the 5d point of view we have

$$S_5 \sim N_c^2 V_5 \int d^5x \sqrt{-g_5} \left(R_5 - 2\Lambda + \frac{1}{\lambda^{3/2}} R_5^4 + \dots - \frac{V_{p-4} N_f}{V_5 N_c} \lambda^{(p-3)/4} - \frac{V_{p-4} N_f}{V_5 N_c} \lambda^{(p-7)/4} R_5^2 + \dots \right). \quad (\text{B.5})$$

In imposing various constraints, first we require

$$V_5 \gg \frac{1}{\lambda^{5/4}}, \quad V_{p-4} \gg \frac{1}{\lambda^{(p-4)/4}}, \quad \lambda \gg 1, \quad . \quad (\text{B.6})$$

for the derivative expansion to be sensible. Next from the above action, we see that for the R^2 terms to produce the leading curvature corrections, we must have

$$1 \gg \frac{V_{p-4} N_f}{V_5 N_c} \lambda^{(p-7)/4} \gg \frac{1}{\lambda^{3/2}}, \quad \text{or} \quad \lambda^{(7-p)/4} \gg \frac{V_{p-4} N_f}{V_5 N_c} \gg \lambda^{(1-p)/4}. \quad (\text{B.7})$$

We also require that the brane tension does not produce a large modification to the cosmological constant (e.g., change the sign of Λ) which gives

$$\lambda^{(3-p)/4} \gg \frac{V_{p-4} N_f}{V_5 N_c}. \quad (\text{B.8})$$

The latter replaces the first inequality in (B.7) giving a more stringent constraint. The second inequality in (B.7) yields

$$\lambda \gg \left(\frac{V_5 N_c}{V_{p-4} N_f} \right)^{\frac{4}{p-1}} \implies \frac{\lambda}{N_c} \gg \left(\frac{V_5}{V_{p-4}} \frac{1}{N_f} \right)^{\frac{4}{p-1}} N_c^{\frac{5-p}{p-1}}. \quad (\text{B.9})$$

If only string loop effects, unsuppressed by powers of ℓ_s , were present then these would dominate if $p \leq 9$. Thankfully, supersymmetry prevents string loop corrections to the lowest order DBI terms and hence this situation does not arise [61].

Finally this formal analysis treats the back-reaction of the D-branes perturbatively and so we must insist on weak string coupling, $g_s \ll 1$. Hence we require that

$$\frac{\lambda}{N_c} \ll 1 \quad (\text{B.10})$$

Comparing to (B.9), we must then have $p > 5$. In other words, only for $p = 7, 9$ can the R^2 term be viewed sensibly as arising from a probe brane and dominating the ℓ_s^6 terms. Further, however, the D9 brane case is not viable since the zero-temperature limit is not supersymmetric. Thus it appears that we can only use D7-branes as sources for the R^2 term in a perturbative setting. Typically there are also couplings of the type

$$T_p \int C \wedge e^F \wedge [\text{tr}(R_T \wedge R_T) - \text{tr}(R_N \wedge R_N)], \quad (\text{B.11})$$

where N and T denote the normal and tangent bundles respectively. One might worry that such couplings would change C and $\mathcal{O}(\ell_s^4)$ and hence feedback at the same order in the Einstein equations. Fortunately for the case of interest this does not happen as can be explicitly checked. The modification only occurs at $\mathcal{O}(\ell_s^8)$ which can be ignored. In 5d-language this translates into ignoring ℓ_s^4 modifications to A_μ .

The next important question to fix is the sign of the R^2 term. Following the general strategy discussed in section 2, we can construct an effective theory in five dimensions for which the coefficient of the R^2 correction is fixed by the trace anomaly of the gauge theory. As in [30], flux terms arising at this order will be ignored. It is interesting then that

scattering amplitudes in string theory from D-branes and O-planes seem to indicate that the sign is positive [62].¹⁰

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